A–3–5. Obtain the transfer functions $E_o(s)/E_i(s)$ of the bridged T networks shown in Figures 3–24(a) and (b).

Solution. The bridged *T* networks shown can both be represented by the network of Figure 3–25(a), where we used complex impedances. This network may be modified to that shown in Figure 3–25(b).

In Figure 3–25(b), note that

$$I_1 = I_2 + I_3, \qquad I_2 Z_1 = (Z_3 + Z_4) I_3$$

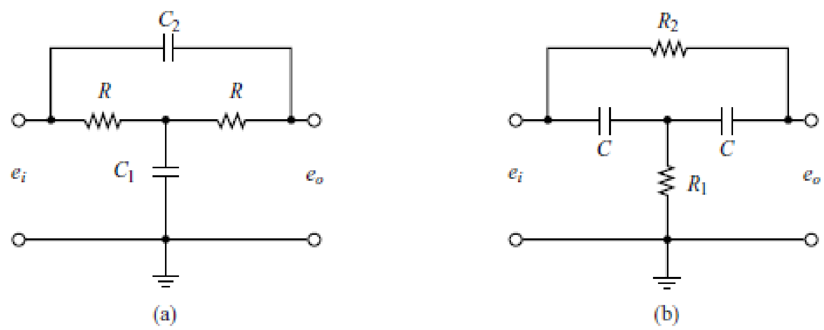


Figure 3–24
Bridged *T* networks.

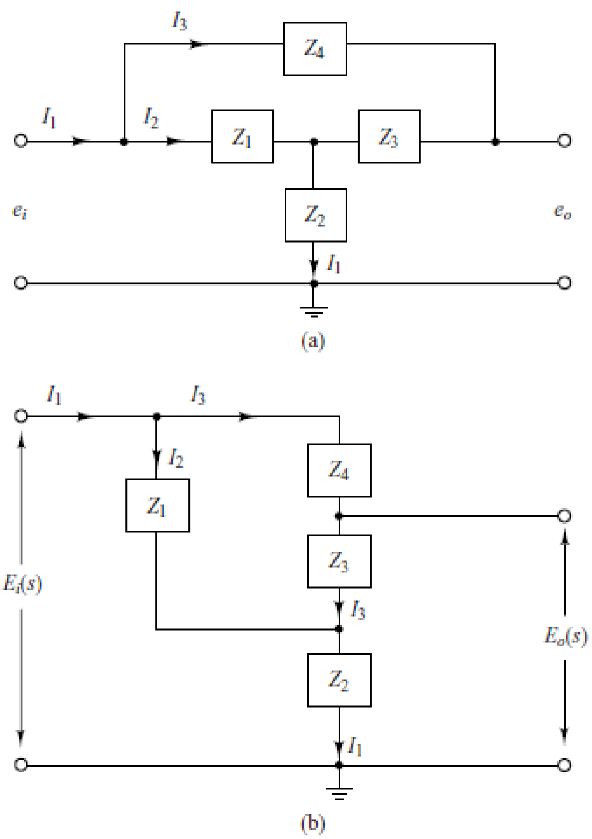


Figure 3–25
(a) Bridged *T* network in terms of complex impedances; (b) equivalent network.

Hence

$$I_2 = \frac{Z_3 + Z_4}{Z_1 + Z_3 + Z_4} I_1, \qquad I_3 = \frac{Z_1}{Z_1 + Z_3 + Z_4} I_1$$

Then the voltages $E_i(s)$ and $E_o(s)$ can be obtained as

$$\begin{aligned} E_i(s) &= Z_1 I_2 + Z_2 I_1 \\ &= \left[Z_2 + \frac{Z_1(Z_3 + Z_4)}{Z_1 + Z_3 + Z_4} \right] I_1 \\ &= \frac{Z_2(Z_1 + Z_3 + Z_4) + Z_1(Z_3 + Z_4)}{Z_1 + Z_3 + Z_4} I_1 \end{aligned}$$

$$\begin{aligned} E_o(s) &= Z_3 I_3 + Z_2 I_1 \\ &= \frac{Z_3 Z_1}{Z_1 + Z_3 + Z_4} I_1 + Z_2 I_1 \\ &= \frac{Z_3 Z_1 + Z_2(Z_1 + Z_3 + Z_4)}{Z_1 + Z_3 + Z_4} I_1 \end{aligned}$$

Hence, the transfer function $E_o(s)/E_i(s)$ of the network shown in Figure 3–25(a) is obtained as

$$\frac{E_o(s)}{E_i(s)} = \frac{Z_3 Z_1 + Z_2(Z_1 + Z_3 + Z_4)}{Z_2(Z_1 + Z_3 + Z_4) + Z_1 Z_3 + Z_1 Z_4} \tag{3–38}$$

For the bridged T network shown in Figure 3–24(a), substitute

$$Z_1 = R, \qquad Z_2 = \frac{1}{C_1 s}, \qquad Z_3 = R, \qquad Z_4 = \frac{1}{C_2 s}$$

into Equation (3–38). Then we obtain the transfer function $E_o(s)/E_i(s)$ to be

$$\begin{aligned} \frac{E_o(s)}{E_i(s)} &= \frac{R^2 + \frac{1}{C_1 s}\left(R + R + \frac{1}{C_2 s}\right)}{\frac{1}{C_1 s}\left(R + R + \frac{1}{C_2 s}\right) + R^2 + R\frac{1}{C_2 s}} \\ &= \frac{RC_1 RC_2 s^2 + 2RC_2 s + 1}{RC_1 RC_2 s^2 + (2RC_2 + RC_1)s + 1} \end{aligned}$$

Similarly, for the bridged T network shown in Figure 3–24(b), we substitute

$$Z_1 = \frac{1}{Cs}, \qquad Z_2 = R_1, \qquad Z_3 = \frac{1}{Cs}, \qquad Z_4 = R_2$$

into Equation (3–38). Then the transfer function $E_o(s)/E_i(s)$ can be obtained as follows:

$$\begin{aligned} \frac{E_o(s)}{E_i(s)} &= \frac{\frac{1}{Cs}\frac{1}{Cs} + R_1\left(\frac{1}{Cs} + \frac{1}{Cs} + R_2\right)}{R_1\left(\frac{1}{Cs} + \frac{1}{Cs} + R_2\right) + \frac{1}{Cs}\frac{1}{Cs} + R_2\frac{1}{Cs}} \\ &= \frac{R_1 C R_2 C s^2 + 2R_1 C s + 1}{R_1 C R_2 C s^2 + (2R_1 C + R_2 C)s + 1} \end{aligned}$$

B–3–8. Consider the electrical circuit shown in Figure 3–37. Obtain the transfer function $E_o(s)/E_i(s)$ by use of the block diagram approach.

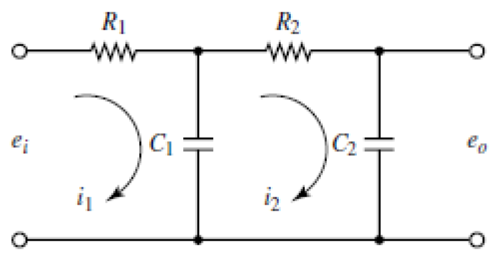


Figure 3–37 Electrical circuit.

B-3-8. Equations for the circuit are

$$\frac{1}{C_1}\int(i_1-i_2)\,dt + R_1 i_1 = e_i$$

$$\frac{1}{C_1}\int(i_2-i_1)\,dt + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt = 0$$

$$\frac{1}{C_2}\int i_2\,dt = e_o$$

The Laplace transforms of these three equations, with zero initial conditions, are

$$\frac{1}{C_1 s}\left[I_1(s) - I_2(s)\right] + R_1 I_1(s) = E_i(s) \qquad (1)$$

$$\frac{1}{C_1 s}\left[I_2(s) - I_1(s)\right] + R_2 I_2(s) + \frac{1}{C_2 s} I_2(s) = 0 \qquad (2)$$

$$\frac{1}{C_2 s} I_2(s) = E_o(s) \qquad (3)$$

Equation (1) can be rewritten as

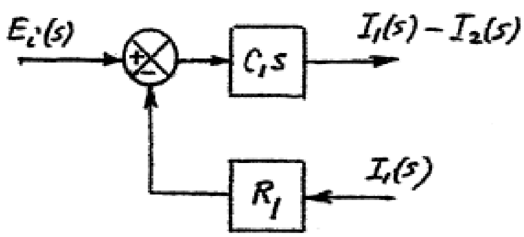
$$C_1 s\left[E_i(s) - R_1 I_1(s)\right] = I_1(s) - I_2(s) \qquad (4)$$

Equation (4) gives the block diagram shown in Figure (a). Equation (2) can be modified to

$$I_2(s) = \frac{C_2 s}{R_2 C_2 s + 1}\,\frac{1}{C_1 s}\left[I_1(s) - I_2(s)\right] \qquad (5)$$

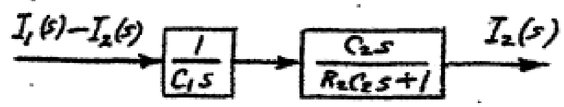
Equation (5) yields the block diagram shown in Figure (b). Also, Equation (3) gives the block diagram shown in Figure (c). Combining the block diagrams of Figures (a), (b), and (c), we obtain Figure (d). This block diagram can be succesively modified as shown in Figures (e) through (h). In this way, we can obtain the transfer function $E_o(s)/E_i(s)$ of the given circuit.

(a)



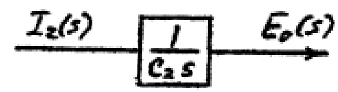
$E_i(s)$
$C_1 s$
$I_1(s) - I_2(s)$
R_1
$I_1(s)$

(b)



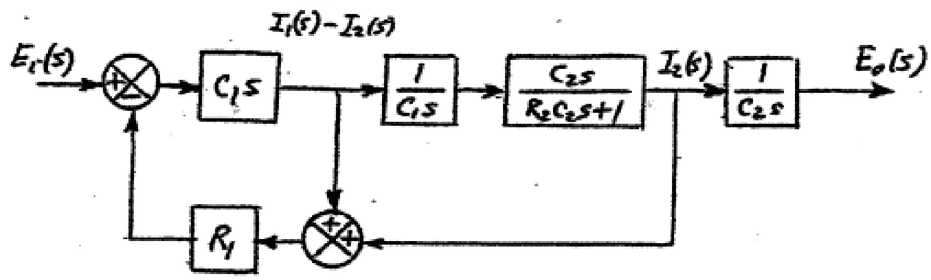
$I_1(s) - I_2(s)$
$\frac{1}{C_1 s}$
$\frac{C_2 s}{R_2 C_2 s + 1}$
$I_2(s)$

(c)



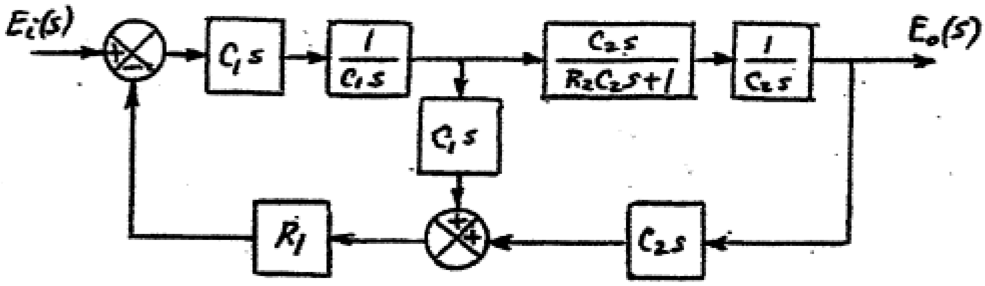
$I_2(s)$
$\frac{1}{C_2 s}$
$E_o(s)$

(d)

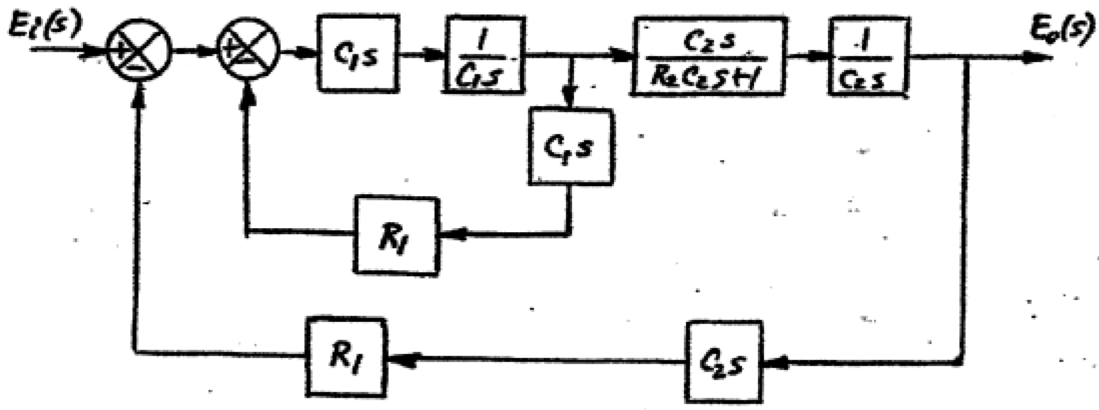


$I_1(s) - I_2(s)$
$E_i(s)$
$C_1 s$
$\frac{1}{C_1 s}$
$\frac{C_2 s}{R_2 C_2 s + 1}$
$I_2(s)$
$\frac{1}{C_2 s}$
$E_o(s)$
R_1

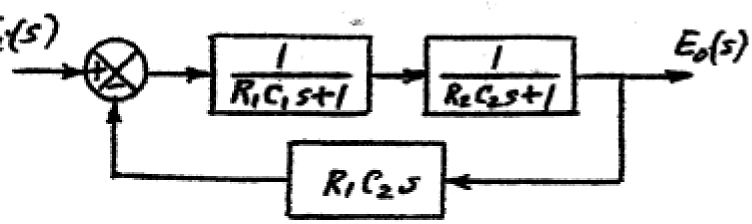
(e)



(f)



(g)



Can be reduced to:

(h)